Math 218 - Final (Spring 2011)

Common Exam - Version A

i- You must show your work to receive credit. Correct answers with inconsistent work will receive no credit.

ii- This is a closed book exam and no calculators are allowed.

iii- Please turn off your cell phone.

iv- The exam is out of 100.

v- There are 7 questions in total. Most questions have several parts to them. Make sure you attempt them all.

vi- Write you section number (see the table below) on the coloured booklet containing your answers, **as well as** the version of the exam you are solving (A or B). Failure to write any of the above shall result in **10 points** being subtracted from your mark.

vii- **DO NOT LOOK AT THE QUESTIONS UNTIL TOLD TO DO SO**

Time	Instructor	Section Number
TR , 12:30 – 13:45	Rana Nassif	1
TR , 11:00 – 12:15	Rana Nassif	2
MWF , 10:00 – 10:50	Sara Abu Diab	3
MWF , 11:00 – 11:50	Monique Azar	4
MWF , 08:00 – 08:50	Friedmann Brock	5
MWF , 13:00 – 13:50	Michella Bou Eid	6
MWF , 14:00 – 14:50	Michella Bou Eid	7
MWF , 09:00 – 09:50	Faleh Taha	8
TR , 09:30 – 10:45	Rana Nassif	9
MWF , 12:00 – 12:50	Monique Azar	10
MWF , 14:00 – 14:50	Tamer Tlas	11

1. Consider the map $T : P_2 \to \mathbb{R}^2$ given by the following formula

$$T(p(x)) = \begin{pmatrix} \int_0^1 p(x)dx \\ p(0) \end{pmatrix}$$

Answer the following questions:

a- (5 points) Is T a linear transformation?

b- (6 points) What is the null space of T?

c- (5 points) Is T one to one? Onto? Invertible?

2. (15 points) Define the inner product on P_2 via $\langle p, q \rangle = \int_1^2 p(x)q(x)dx$. Find an orthonormal basis for P_2 starting from its standard one (i.e. starting from $\{1, x, x^2\}$

3. Consider the following matrix

$$A = \begin{pmatrix} -1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

a- (10 points) Find its eigenvalues and the corrseponding eignespaces. State the geometric and algebraic multiplicities of each eigenvalue.

b- (5 points) Is there a matrix P which satisfies $D = P^{-1}AP$ where D is the diagonal matrix whose entries are the eigenvalues of A? If yes, write this matrix down. If no, state the reason why it doesn't exist.

4. (9 points each) Prove the following two facts:

a- Let V be a vector space with an inner product, and let W be a vector subspace of V. Let u be a vector in V. Prove that $||\text{proj}_W(u)|| \leq ||u||$, where $\text{proj}_W(u)$ is the projection of u onto W.

b- Let V be a vector space with an inner product, and let W be a vector subspace of V. Prove that $\{W^\perp\}^\perp = W$.

5. Consider the following:

a- (6 points) For which values of k does the following system has exactly one solution?

$$\begin{aligned} kx + z &= 3 \\ y - z &= 2 \\ z &= 1 \end{aligned}$$

b- (5 points) If $k = 0$ how many solutions does the system have?

6. (15 points) Find the line which best fits the following data

x	y
1	0
1	2
0	0

7. (10 points) Let V be the subspace of $\mathbb{R}^3$ spanned by the following collection of vectors $\left\{ \begin{pmatrix} 2 \\ -3 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 2 \\ 2 \end{pmatrix}, \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix}, \begin{pmatrix} 4 \\ 0 \\ 4 \end{pmatrix} \right\}$. Find a basis for V.